Reading 6

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PARALLEL CIRCUITS

When two or more components are connected across one voltage source they form a parallel circuit. The two lamps in figure 1 are in parallel with each other and with the battery. Each parallel path is called a branch, with its own individual current. Parallel circuits have one common voltage across all the branches but the individual branch currents can be different.

The voltage is the same across all components in a parallel circuit.

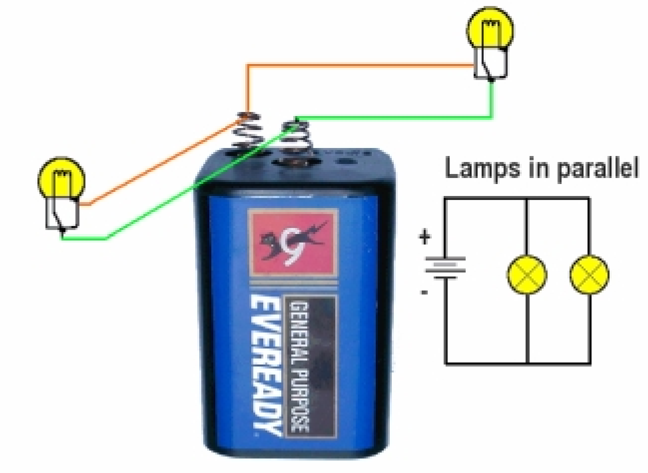


Figure 1.

In figure 1 (pictorial diagram above and the equivalent schematic circuit to the right), the two lamps are actually *directly* connected to the battery terminals. This is always the case with parallel circuits. If you had 10 components (they don't have to be lamps) connected in parallel then each side of each component is connected directly to the battery (or other source).

BRANCH CURRENTS

Each resistance (or other components) in a parallel circuit is connected by a conductor directly to the source voltage. Each resistor will draw current from the source according to Ohm's law, I=E/R, for each branch. The sum of all the branch currents must then be equal to the total current drawn from the source.

The sum of the branch currents equals the total current.

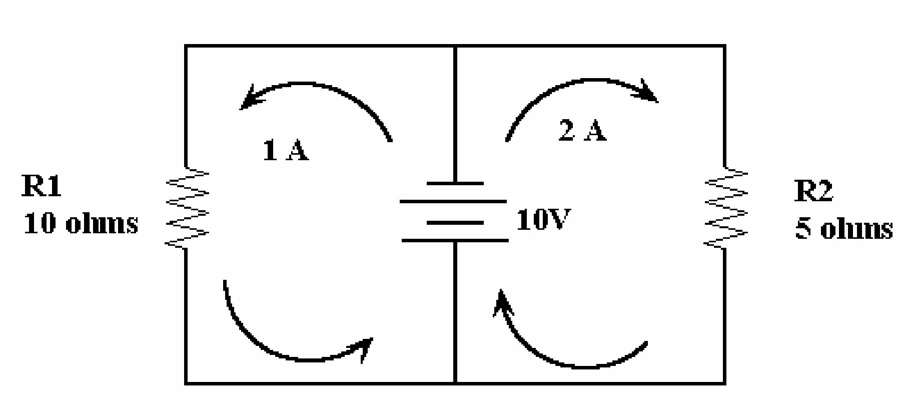


Figure 2.

In applying Ohm's law, it is important to note that the current equals the voltage applied across the circuit divided by the resistance between the two points where that voltage is applied.

In figure 2, 10 V is applied across the 10Ω of R_1, resulting in a current of 1 ampere being drawn from the battery through R_1. Similarly the 10 volts applied to the 5Ω of R_2 will cause 2 amperes to be drawn from the battery.

The two branch currents in the circuit are then 1 ampere and 2 amperes. The total current drawn from the battery is then 3 amperes.

Just as in a circuit with only one resistance, any branch that has less resistance will draw more current. If R_1 and R_2 were equal however, the two branch currents would have the same value. For instance, if R_1 and R_2 were both 5Ω then each branch would draw 2 amperes and the total current drawn from the battery would be 4 amperes.

The current can be different in parallel circuits having different resistances because the voltage is the same across all the branches. Any voltage source generates a potential difference across its two terminals. This voltage does not move. Only current flows around the circuit. The source voltage is available to make electrons move around any closed path connected to the generator terminals. How much current is in the separate paths depends on the amount of resistance in each branch.

For a parallel circuit with any number of branch currents we can then write an equation for calculating the total current (I_t):

$$I_t = I_1 + I_2 + I_3 + I_4 \text{ etc.}$$

This rule applies for any number of parallel branches, whether the resistances are equal or unequal.

Example:

An R_1 of 20 Ω and an R_2 of 40 Ω and an R_3 of 60 Ω are connected in parallel across a 240 volt supply. What is the total current drawn from the supply?

Let's calculate the branch currents for R_1, R_2 and R_3:

$I_{R1} = E/R_1 = 240/20 = 12$ Amps

$I_{R2} = E/R_2 = 240/40 = 6$ Amps

$I_{R3} = E/R_3 = 240/60 = 4$ Amps

The total current drawn from the 240 volt supply is the sum of the branch currents:

$I_t = I_{R1} + I_{R2} + I_{R3} = 12 + 6 + 4 = 22$ Amperes.

RESISTANCES IN PARALLEL

In the example above we could have worked out the total resistance in order to calculate the total current being drawn from the supply.

To find the total resistance of any number of resistors in parallel we find the reciprocal of the sum of the reciprocals for each resistance. This sounds like a bit of a mouthful so I will put it in equation form and you should see what I mean.

$$R_t = \frac{1}{1/R_1 + 1/R_2 + 1/R_3}$$

Let's calculate the total resistance of our example using this equation.
Firstly, find the reciprocal of each of the resistances:

Reciprocal of $R_1 = 1/R_1 = 1/20 = 0.05$
Reciprocal of $R_2 = 1/R_2 = 1/40 = 0.025$
Reciprocal of $R_3 = 1/R_3 = 1/60 = 0.01667$ (recurring decimal).

The sum of the reciprocals above is: 0.091667

Finally, to find R_t we take the reciprocal of the sum of the reciprocals, or:

1/0.091667 which equals 10.91 rounded.

So $R_t = 10.91$ ohms (with a little rounding error).

Since we know the total resistance and the applied voltage we can now calculate the total current from Ohm's law.

$I=E/R = 240/10.91 = 21.998$ Amps, or close enough to the 22 Amps we calculated earlier. When you get used to it these calculations are very easy to do on a calculator. Many calculators have a reciprocal key '1/x'.

Example: A parallel circuit consisting of two branches, each with a 5 A current, is connected across a 90 V source. How much is the equivalent total resistance R_t?

To find the total resistance (R_t) we need to know the applied voltage (90 V) and the total current drawn from the supply. Since we are told that there are two branches and each branch draws 5 A then the total current must be 10 A. We can now use Ohm's Law to calculate the total resistance:

R=E/I
R=90/10
R=9 Ω

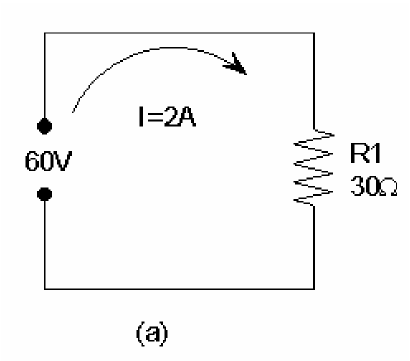
PARALLEL BANK

A combination of parallel branches is often called a bank. Radio operators often use a device called a dummy load (we will go into detail about dummy loads later). A dummy load is typically just a resistance of 50 ohms which is connected to the antenna socket of a transmitter for tuning purposes. The problem is the dummy load has to dissipate all of the transmitter power. If the operator purchased and used a 50 ohm carbon resistor it would burn out, as carbon resistors cannot dissipate more than about 1 W. A typical dummy load should be able to dissipate 100 W.

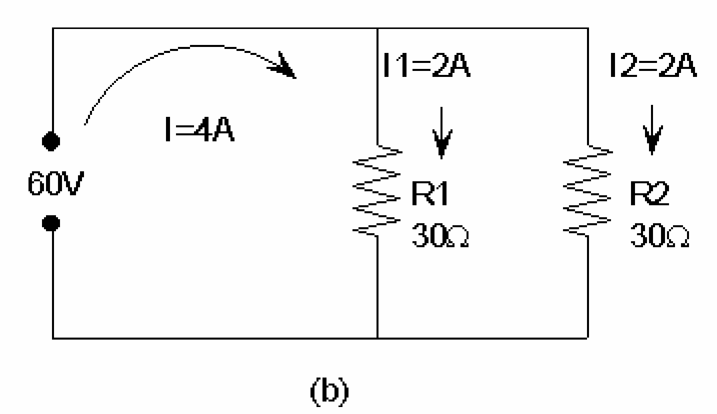
A good dummy load can be made from twenty 1000Ω 5 Watt resistors connected in parallel. Each of the twenty resistors, being 5 watt each are able to dissipate 5 watts, so the bank is able to dissipate 20 x 5 = 100 watts. The total resistance of twenty 1000 Ω resistors in parallel is 50 Ω with a total power rating of 100 Watts.

In practice the dummy load is usually cooled, perhaps by immersing the resistors in oil or some sort of air cooling which enables the resistors to dissipate more power without over heating and being destroyed.

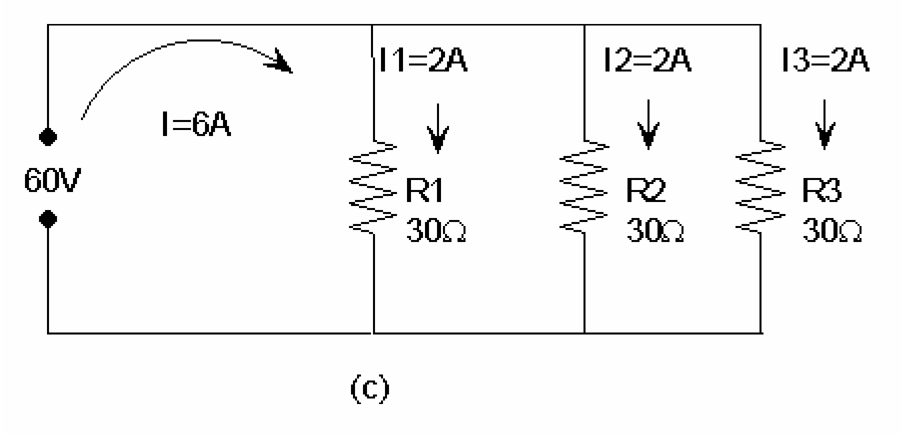
A stumbling block for many is trying to understand how adding more resistance to a parallel circuit can actually reduce the total resistance.



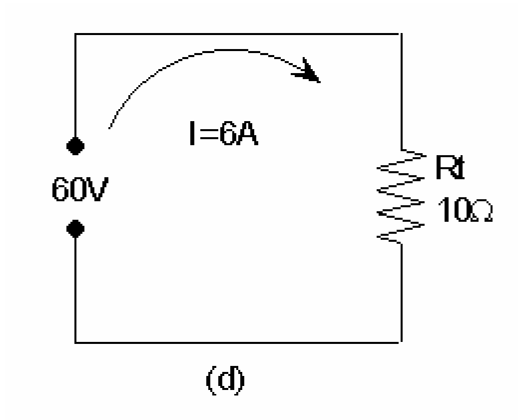
In figure 3(a) you see a supply voltage of 60 volts connected to a 30 Ω load. By Ohm's law the current drawn by the 30 Ω load must be 2 A.



If a second 30 Ω resistor is now connected as in figure 3(b), then an additional 2 A will flow through it. The total current drawn from the supply is now 4 A.



Adding a third 30 Ω resistor as in figure 3(c) causes a further 2 A to be drawn from the supply, bringing the total current drawn from the supply to 6 A.



The total resistance of this circuit would be:
R=E/I = 60/6 = 10 Ω

Figures 3(a)-(d).

We can see that as we add parallel resistances to the supply, more current is drawn from the supply, so the total circuit resistance must be less. We could go on adding parallel resistances for as long as we wanted, and each time we did, the total circuit resistance would become less with each added resistance.

DERIVING THE RECIPROCAL EQUATION

We discussed the reciprocal equation earlier but we did not mention how this equation was actually derived.

We know the basic law: the sum of the branch currents is equal to the total current flowing in a parallel circuit.

$$I_t = I_1 + I_2 + I_3 \text{ etc.}$$

$$I_t = E/R_t \text{ and } I_1 = E/R_1 \text{ and } I_2 = E/R_2 \text{ etc.}$$

We can substitute this into the rule for branch currents and get:

$$E/R_t = E/R_1 + E/R_2 + E/R_3 \text{ etc.}$$

Notice how E is in the numerator on both sides of the equal sign. If we divide both sides by E, the E's cancel out and we are left with:

$1/R_t = 1/R_1 + 1/R_2 + 1/R_3$ etc.

This gives us the reciprocal of the total resistance. The reciprocal of this is the total resistance.

SPECIAL CASE OF ALL RESISTANCES THE SAME

If all of the resistances are the same in a parallel circuit, as in our dummy load example presented earlier, the total resistance can be found by dividing the number of branches into the value of one of the parallel resistances.

In the dummy load example we had 20, 1000 Ω resistors. To find the total resistance a quick method is to divided 1000/20 = 50 Ω. However, please do remember that this method can only be used when all resistances are the same.

SPECIAL CASE OF ONLY TWO BRANCHES.

When there are two parallel resistances and they are not equal, it is usually quicker to calculate the combined resistance by the method known as product over sum. This rule says that the combination of two parallel resistances is their product divided by their sum.

$R_t = (R_1 \times R_2) / (R_1 + R_2)$

The symbol '/' means divided by.

Example:

A resistance of 100 Ω is connected in parallel with a resistance or 200 Ω. What is the total resistance of the circuit using the product over sum method.

$R_t = (100 \times 200) / (100 + 200)$
$R_t = 20000 / 300$
$R_t = 66.667\ \Omega$ (rounded).

Notice also that the total resistance of parallel resistors is always less than the lowest resistance.

TOTAL POWER IN PARALLEL CIRCUITS

Since the power dissipated in the branch resistances must come from the source voltage, the total power equals the sum of the individual values of power in each branch.

$P_t = P_{R1} + P_{R2} + P_{R3}$ etc.

Of course if you know the total current and the applied voltage, you can find the power by P = EI.

I can recall a question that was asked in a Novice exam some years ago. I am sure the question was not meant to be as difficult as it turned out and my guess is that the person writing the question must have been anxious to knock off work at the time.

The question was:

> Four 10Ω, 1 watt resistors are connected in series. What is the total power dissipation rating of the circuit?

That was it, no voltage or current mentioned. In fact the resistance (10 Ω) is totally irrelevant as is the series connection.

The answer is 4 watts.

The question was badly phrased in my view and could have been put:

> What is the maximum power that four, 1 watt resistors can dissipate?

ADVANTAGE OF PARALLEL CONNECTION

Household appliances, as you may know, are designed to operate at 240 V. The electrical wiring to all the power points and light fittings in your home are in the form of a parallel circuit. This ensures that each appliance receives the supply voltage of 240 V. Imagine the problems if your house was wired as a series circuit. Each time you turned on a light all the lights in the house would get dimmer. If you turned on something with a high resistance (low wattage) the lights would not even glow. Supplying the same voltage to various components whether it be in a house, a radio circuit, or a car, requires parallel connections.

End of Reading 6.
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